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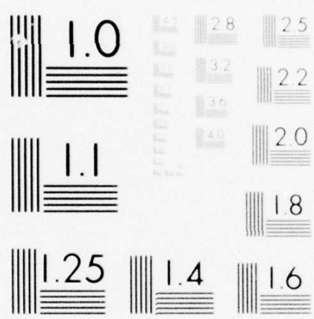

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MOD-ANOVA: AN ANALYSIS OF VARIANCE PROGRAM FOR 2^n AND 3^n FACTORIAL EXPERIMENTS

JULY 1977

PREPARED BY

~~METHODOLOGY, RESOURCES, AND COMPUTATION DIRECTORATE~~

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Modulo Analysis of Variance (MOD-ANOVA) is a computer program for 2^n and 3^n full or fractional factorial experiments. The program applies modular arithmetic to cell identification numbers to classify and sum the observations. MOD-ANOVA is fast and efficient and is extremely simple in structure. The complete program contains less than 130 lines of code. The program is written in FORTRAN V and is operational on the UNIVAC 1108 computer, but it should be transferable to any computer compatible with FORTRAN.

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MOD-ANOVA: An Analysis of Variance Program

for 2^n and 3^n Factorial Experiments

ABSTRACT

Modulo Analysis of Variance (MOD-ANOVA) is a computer program for the analysis of 2^n or 3^n factorial experiments. The program is applicable for either full or fractional factorial designs. The program applies modular arithmetic to the cell identification numbers to classify and sum observations. MOD-ANOVA is fast, efficient, and extremely simple in structure. The complete program contains less than 130 lines of code. The program, written in FORTRAN V, is operational on the UNIVAC 1108 computer, but it should be transferable to any computer compatible with FORTRAN V.

MOD-ANOVA: An Analysis of Variance Program for 2^n and 3^n Factorial Experiments

CONTENTS

TABLES

MOD-ANOVA. An Analysis of Variance Program for 2^n and 3^n Factorial Experiments

1. INTRODUCTION

Modulo Analysis of Variance (MOD-ANOVA) is a computer program for performing the analysis of variance computations for a fractional factorial design. The program is applicable for 2^n and 3^n designs. It is, to the knowledge of the authors, unique in its application of modular arithmetic. The program originated from a requirement for the analysis of variance of a one-ninth fractional factorial of a 3^7 design. Although the program was developed for fractional 3^n designs, it is also applicable to fractional 2^n designs and full 2^n and 3^n designs.

A common procedure for performing ANOVA computations for fractional 2^n and 3^n designs is to perform an ANOVA on a lower order full design and then use the aliases to associate sum of squares with the proper factorial effects. This works well for 2^n fractional designs, but the procedure becomes extremely tedious for 3^n fractional designs as n becomes large. An alternative method for performing the computations is to employ the General Linear Hypothesis Model and apply least squares. For large designs this involves the construction of extensive design matrices and the inversion of many high order matrices. Because of the above cited criticism of existing computational methods, an alternative computational scheme was sought. This report documents the computational procedure ultimately developed.

Advantages of the procedure are its simplicity and computational efficiency.

2. BACKGROUND

Theory of fractional factorials for 2^n and 3^n designs was developed by Finney (1945) and (1946). The interested reader is referred to the more readily available references of Kempthorne (1952), Davies (1960), Winer (1962), or Kirk (1968). Kempthorne is more theoretical than the other references and uses finite group theory in the development of fractional factorials. Cochran and Cox (1957), Connor and Zelen (1957) and (1959), and Davies (1960) contain fractional factorial designs for 2^n and 3^n experiments. (For References, See Appendix B.)

3. NOTATION AND DEFINITIONS

Consider an m^n factorial experiment, where n is the number of factors and m is the number of levels of each factor and is restricted to either 2 or 3. Denote the general cell identification by lower case letters abc ..., and denote factorial effects by upper case letters A,B,C,..... For 2^n designs, lower case letters may take on 0 or 1; for 3^n designs, they may take on 0,1, or 2. A fractional replicate involving a subset of m^{n-r} factor level combinations of the full design is termed a $(1/m^r)$ replicate. The total degrees of freedom of a full design is $m^n - 1$; the total degrees of freedom of a $(1/m^r)$ replicate design is $(m^n - 1)/m^r$.

Let I be the usual identity element. The defining contrast of a $(1/2^r)$ replicate consists of I equated to $(2^r - 1)$ groups of letters

$A^{a_{11}} B^{a_{21}} C^{a_{31}}$ ..., where $a_{1j}, {}_{2j}, a_{3j}$... take on the values of 0 or 1 and $A^0 = B^0 = ... = 1$. For a $(1/3^r)$ replicate, I is equated to $(3^r - 1)/2$ groups of letters and a_{1j}, a_{2j}, a_{3j}... take on 0, 1 or 2. (See Connor and Zelen (1957) and (1958).) The conventional modular notation is used for expressing interaction components. Modular notation is used by Kempthorne (1952), Winer (1962), Hicks (1964), and Kirk (1968) and readily lends itself to extension and generalization. Winer (1962) and Kirk (1968) give equivalences between modular notation and the older Yates (1937) notation. For the reader familiar with the Yates notation, the following equivalences are given.

<u>Modular notation</u>	<u>Yates notation</u>
AB	AB(J)
AB^2	AB(I)
ABC	ABC(Z)
ABC^2	ABC(Y)
AB^2C	ABC(X)
AB^2C^2	ABC(W)

Conventionally the exponent of the first letter in the group of letters is always made unity since it can be shown, for example, $A^2BC = (A^2BC)^2 = A^4B^2C^2 = AB^2C^2$ modulo 3. Each component of the 3^n designs has two degrees of freedom. Therefore, $(m^n - 1)/2m^r$ components can be partitioned from the total sum of squares of a $(1/3^r)$ replicate.

4. COMPUTATIONAL PROCEDURE

The computational procedure of MOD-ANOVA is based on a linear function comparable to Kempthorne's rule (1952) for confounding. Effects or components of effects in an ANOVA model (of a full design or after fractionating) are identified. The program then computes the sum of squares for each effect identified. For $m = 2$, the sums of squares are for whole effects; for $m = 3$, the sums of squares are for whole main effects or interaction components. In the latter case, each sum of squares has two degrees of freedom.

Consider the linear function

$$L = a_{1j}x_1 + a_{2j}x_2 + a_{3j}x_3 + \ldots + a_{nj}x_n;\ j = 1,2,\ldots,h, \qquad (1)$$

where the a_{ij} coefficients are as defined in Section 3 above and h is the number of effects to be calculated for the ANOVA. The variables (x_i; $i = 1,2,\ldots,n$) represent the ABC... also defined in Section 3 above. The L-function is applied to each abc.... cell of the design for each effect to be calculated. This results in a partitioning of the cells into m parts, each part having an equal number of cells. That is, evaluate

$$L = b \pmod{m};\ b = 0,1,\ldots, m-1. \qquad (2)$$

For example, if $m = 2$ we have

$$L = 0 \pmod 2$$

and

$$L = 1 \pmod 2.$$

If $m = 3$, we have

$$L = 0 \pmod 3,$$

$$L = 1 \pmod 3,$$

and

$$L = 2 \pmod 3.$$

The observations (y_k's) within each of the m partitions are then summed, squared, and divided by the number of the observations which went into the sum. For example, consider the ABC^2 component. Then, $L = 1x_1 + 1x_2 + 2x_3$. For $L = 0$, 1, and 2, denote the three sums by $(ABC^2)_0$, $(ABC^2)_1$, and $(ABC^2)_2$, respectively. Denote the three sums squared divided by the number of observations by $(ABC^2)_0^2$, $(ABC^2)_1^2$, and $(ABC^2)_2^2$. The sum of squares due to the interaction component ABC^2 is then

$$SS|ABC^2| = (ABC^2)_0^2 + (ABC^2)_1^2 + (ABC^2)_2^2 - CF, \qquad (3)$$

where CF is the usual correction factor. The sum of squares due to the ABC interaction is the sum of the four components' sum of squares

$$SS|ABC| = SS|ABC| + SS|ABC^2| + SS|AB^2C| + SS|AB^2C^2| \quad (4)$$

The above described computations are illustrated in the following numerical example.

5. NUMERICAL EXAMPLE

a. Problem Description. Consider a one-third replicate of a 3^5 factorial experiment. The 243 cell identifications of a full design are shown in Table 1. Suppose the fourth order interaction (ABCDE) is selected for confounding. Selecting the principal block gives the resulting one-third replicate defined by the 81 circled cell identifications. (See Connor and Zelen (1959).) Note that the sum of the cell identifications equal 0 (mod 3). The data for the numerical example are given in Table 2.

The aliases are obtained by using the defining contrast I = ABCDE and the rule of generalized interaction for 3^n systems. Aliases of the main effects and the first order interaction components are given in Table 3. The Effect Column provides the needed coefficients for the L-function of equation (1). Each of the effects defines an L-function. Therefore, we have 25 L-functions enumerated in Table 4 below. The coefficients are the exponents of the Effects.

Table 1. Full Replicate of a 3^5 Factorial Experiment

Levels of the factors			D_0			D_1			D_2		
			E_0	E_1	E_2	E_0	E_1	E_2	E_0	E_1	E_2
A_0	B_0	C_0	00000	00001	00002	00010	00011	00012	00020	00021	00022
		C_1	00100	00101	00102	00110	00111	00112	00120	00121	00122
		C_2	00200	00201	00202	00210	00211	00212	00220	00221	00222
	B_1	C_0	01000	01001	01002	01010	01011	01012	01020	01021	01022
		C_1	01100	01101	01102	01110	01111	01112	01120	01121	01122
		C_2	01200	01201	01202	01210	01211	01212	01220	01221	01222
	B_2	C_0	02000	02001	02002	02010	02011	02012	02020	02021	02022
		C_1	02100	02101	02102	02110	02111	02112	02120	02121	02122
		C_2	02200	02201	02202	02210	02211	02212	02220	02221	02222
A_1	B_0	C_0	10000	10001	10002	10010	10011	10012	10020	10021	10022
		C_1	10100	10101	10102	10110	10111	10112	10120	10121	10122
		C_2	10200	10201	10202	10210	10211	10212	10220	10221	10222
	B_1	C_0	11000	11001	11002	11010	11011	11012	11020	11021	11022
		C_1	11100	11101	11102	11110	11111	11112	11120	11121	11122
		C_2	11200	11201	11202	11210	11211	11212	11220	11221	11222
	B_2	C_0	12000	12001	12002	12010	12011	12012	12020	12021	12022
		C_1	12100	12101	12102	12110	12111	12112	12120	12121	12122
		C_2	12200	12201	12202	12210	12211	12212	12220	12221	12222
A_2	B_0	C_0	20000	20001	20002	20010	20011	20012	20020	20021	20022
		C_1	20100	20101	20102	20110	20111	20112	20120	20121	20122
		C_2	20200	20201	20202	20210	20211	20212	20220	20221	20222
	B_1	C_0	21000	21001	21002	21010	21011	21012	21020	21021	21022
		C_1	21100	21101	21102	21110	21111	21112	21120	21121	21122
		C_2	21200	21201	21202	21210	21211	21212	21220	21221	21222
	B_2	C_0	22000	22001	22002	22010	22011	22012	22020	22021	22022
		C_1	22100	22101	22102	22110	22111	22112	22120	22121	22122
		C_2	22200	22201	22202	22210	22211	22212	22220	22221	22222

Table 2. One-third Replicate of a 3^5

Levels of the factors			D_0			D_1			D_2		
			E_0	E_1	E_2	E_0	E_1	E_2	E_0	E_1	E_2
A_0	B_0	C_0	1996					1638		1859	
		C_1			1978		2088		2197		
		C_2		2016		2398					2330
	B_1	C_0			1818		1806		1972		
		C_1		1978		2185					2499
		C_2	2636					2838		2382	
	B_2	C_0		2076		2115					2003
		C_1	2460					2591		2655	
		C_2			2792		2649		2562		
A_1	B_0	C_0			1687		1681		1822		
		C_1		2062		2304					2327
		C_2	2384					2581		2122	
	B_1	C_0		1623		1841					1586
		C_1	2012					1942		2368	
		C_2			2203		2345		1996		
	B_2	C_0	1651					1634		1665	
		C_1			1854		2064		2388		
		C_2		2195		2391					1685
A_2	B_0	C_0		1511		1687					1882
		C_1	1877					2379		2462	
		C_2			2081		2288		2077		
	B_1	C_0	1562					1776		2080	
		C_1			1982		2082		2280		
		C_2		2296		2308					1652
	B_2	C_0			1595		1501		1860		
		C_1		2096		2061					1889
		C_2	2165					2007		1625	

Table 3. Aliases

$I = ABCDE$

Effect	Aliases	
	$ABCDE$	$A^2B^2C^2D^2E^2$
A	$AB^2C^2D^2E^2$	$BCDE$
B	AB^2CDE	$ACDE$
C	ABC^2DE	$ABDE$
D	$ABCD^2E$	$ABCE$
E	$ABCDE^2$	$ABCD$
AB	$ABC^2D^2E^2$	CDE
AB^2	$AC^2D^2E^2$	$BC^2D^2E^2$
AC	$AB^2CD^2E^2$	BDE
AC^2	AB^2D^2E	BC^2DE
AD	$AB^2C^2DE^2$	BCE
AD^2	$AB^2C^2E^2$	BCD^2E
AE	$AB^2C^2D^2E$	BCD
AE^2	$AB^2C^2D^2$	$BCDE^2$
BC	AB^2C^2DE	ADE
BC^2	AB^2DE	AC^2DE
BD	AB^2CD^2E	ACE
BD^2	AB^2CE	ACD^2E
BE	AB^2CDE^2	ACD
BE^2	AB^2CD	$ACDE^2$
CD	ABC^2D^2E	ABE
CD^2	ABC^2E	ABD^2E
CE	ABC^2DE^2	ABD
CE^2	ABC^2D	$ABDE^2$
DE	$ABCD^2E^2$	ABC
DE^2	$ABCD^2$	$ABCE^2$

Table 4. Coefficients of L-Functions

Effect	a_1	a_2	a_3	a_4	a_5	Effect	a_1	a_2	a_3	a_4	a_5
A	1	0	0	0	0	BC	0	1	1	0	0
B	0	1	0	0	0	BC^2	0	1	2	0	0
C	0	0	1	0	0	BD	0	1	0	1	0
D	0	0	0	1	0	BD^2	0	1	0	2	0
E	0	0	0	0	1	BE	0	1	0	0	1
AB	1	1	0	0	0	BE^2	0	1	0	0	2
AB^2	1	2	0	0	0	CD	0	0	1	1	0
AC	1	0	1	0	0	CD^2	0	0	1	2	0
AC^2	1	0	2	0	0	CE	0	0	1	0	1
AD	1	0	0	1	0	CE^2	0	0	1	0	2
AD^2	1	0	0	2	0	DE	0	0	0	1	1
AE	1	0	0	0	1	DE^2	0	0	0	1	2
AE^2	1	0	0	0	2						

b. Program Input. The program input consists of a read format for reading cell ID and data, a format for writing a_{ij}'s, b (mod m) sums and the sums of squares, the coefficients of the L-function, cell identifications, and the data. Input format descriptions are given below. Following the description is an illustration of the program input for the above described example.

Card Type	Purpose	Format
1	Format to read cell ID and data	(10A6)
2	Format to write a_{ij}'s, b (mod m) sums and sums of squares	(10A6)
3	Read number of factors, number of levels, fractionating (m^r), and number of effects to be calculated in addition to the main and two factor interactions	(4I5)
4	Cell ID and data values	(According to format on Card Type 1)

Numerical Example Input

```
@RUN,/TP    A106C,F3880A7997B,UNCLASSIFIED,2,200
@ASG,A 06MODANOVA.
@XQT  06MODANOVA.VAR
      (1X,5I1,F9.0)
      (1X,5I1,4X,4F14.4)
   5    3    3
 00000       1996.
 00012       1638.
 00021       1859.
          .
          .
          . (See output for complete data)
          .
          .
          .
 22221       1625.
```

c. Program Output. Program output consists of the input data, coefficients of the L-functions, and their sums of squares, along with the sum of squares of all observations and the correction factor. The difference of the latter two gives the total sum of squares. Output from the above numerical example follows.

```
@RUN,/TP     A106A,F3880A79978,UNCLASSIFIED,7,200

@ASG,A 06MODANOVA.

@XQT  06MODANOVA.VAR
 00000     1996.
 00012     1638.
 00021     1859.
 00102     1978.
 00111     2088.
 00120     2197.
 00201     2016.
 00210     2398.
 00222     2330.
 01002     1818.
 01011     1806.
 01020     1972.
 01101     1978.
 01110     2185.
 01122     2499.
 01200     2535.
 01212     2836.
 01221     2382.
 02001     2076.
 02010     2115.
 02022     2003.
 02100     2460.
 02112     2591.
 02121     2655.
 02202     2792.
 02211     2649.
 02220     2562.
 10002     1687.
 10011     1681.
 10020     1822.
 10101     2062.
 10110     2304.
 10122     2327.
 10200     2384.
 10212     2581.
 10221     2122.
 11001     1623.
 11010     1841.
 11022     1586.
 11100     2012.
 11112     1942.
 11121     2368.
 11202     2203.
 11211     2345.
 11220     1996.
 12000     1651.
 12012     1634.
 12021     1665.
```

12102	1864.
12111	2064.
12120	2388.
12201	2195.
12210	2391.
12222	1685.
20001	1511.
20010	1687.
20022	1882.
20100	1877.
20112	2379.
20121	2462.
20202	2081.
20211	2288.
20220	2077.
21000	1562.
21012	1776.
21021	2080.
21102	1982.
21111	2082.
21120	2280.
21201	2296.
21210	2308.
21222	1652.
22002	1595.
22011	1501.
22020	1860.
22101	2096.
22110	2051.
22122	1889.
22200	2165.
22212	2007.
22221	1625.

ABCDE	0(MOD M)	1(MOD M)	2(MOD M)	SUM OF SQUARES
MEAN				.3479+009
10000	60416.0000	54413.0000	51051.0000	1137460.2222
01000	55714.0000	55947.0000	56219.0000	4732.0741
00100	47927.0000	59050.0000	60903.0000	3648534.7407
00010	54485.0000	57170.0000	56225.0000	137405.5556
00001	57076.0000	55575.0000	55229.0000	71408.9630
11000	54045.0000	55772.0000	58063.0000	300932.5185
12000	53205.0000	58891.0000	55784.0000	600435.6296
10100	56283.0000	54320.0000	57277.0000	167719.1852
10200	55103.0000	56790.0000	55987.0000	52743.6296
10010	55687.0000	55786.0000	56407.0000	11282.0000
10020	56239.0000	56209.0000	55432.0000	15504.6667
10001	55860.0000	55541.0000	56479.0000	16848.9630
10002	55788.0000	57217.0000	54875.0000	103216.9630
01100	56366.0000	55809.0000	55705.0000	9357.8519
01200	55162.0000	56389.0000	56329.0000	35444.6667
01010	55410.0000	55385.0000	57085.0000	70324.0741
01020	55047.0000	56090.0000	56743.0000	54205.8519
01001	55564.0000	54830.0000	57486.0000	139347.8519
01002	55752.0000	56100.0000	56028.0000	2499.5556
00110	58389.0000	52409.0000	57082.0000	732167.6296
00120	53636.0000	56833.0000	57411.0000	306240.9630
00101	55865.0000	55725.0000	56290.0000	6412.9630
00102	56530.0000	55291.0000	56059.0000	28972.6667
00011	57246.0000	54986.0000	55648.0000	99993.1852
00012	54999.0000	56488.0000	56393.0000	51473.8519
R				584209.3333
TOTAL				.3563+09

@FIN

Output from the program gives the required sums of squares to construct the desired analysis of variance table. For example, sums of squares of whole effects equals the sum of the sums of squares of its components. The four-degree of freedom whole effect of the AB interaction is the sum of the two two-degree of freedom components AB and AB^2. That is,

$$SS[AB] = SS[AB] + SS[AB^2].$$

The corrected sum of squares of total is

$$SS[T] = TOTAL - MEAN.$$

The sum of squares of the residual is then

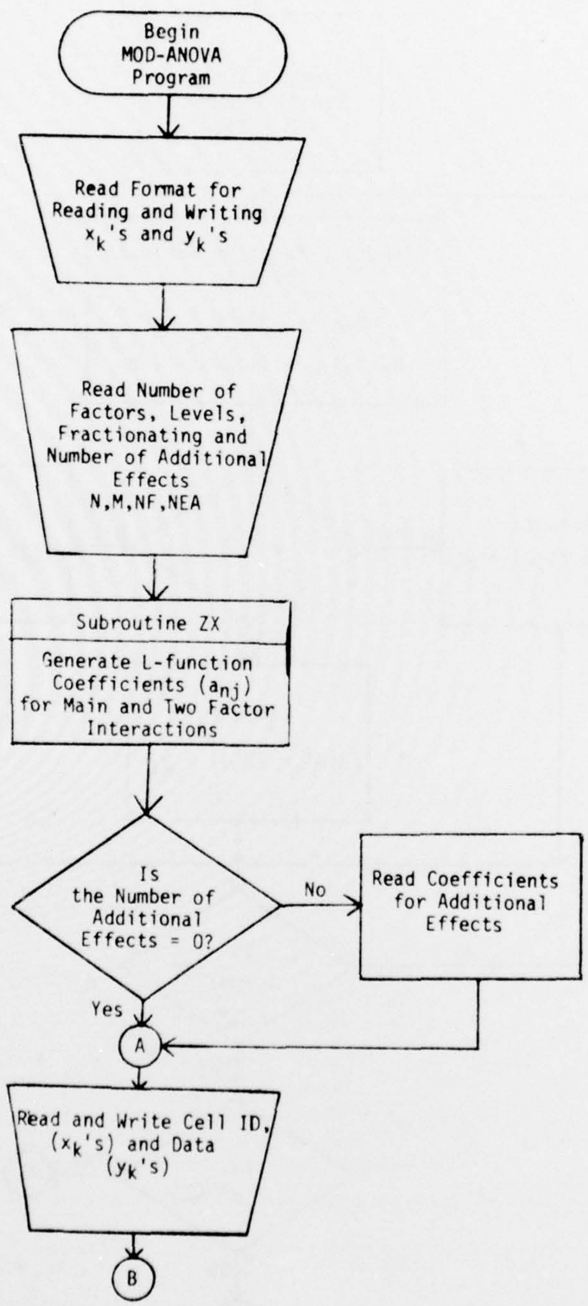
$$SS[R] = SS[T] - \sum (SS \text{ of all fitted effects}).$$

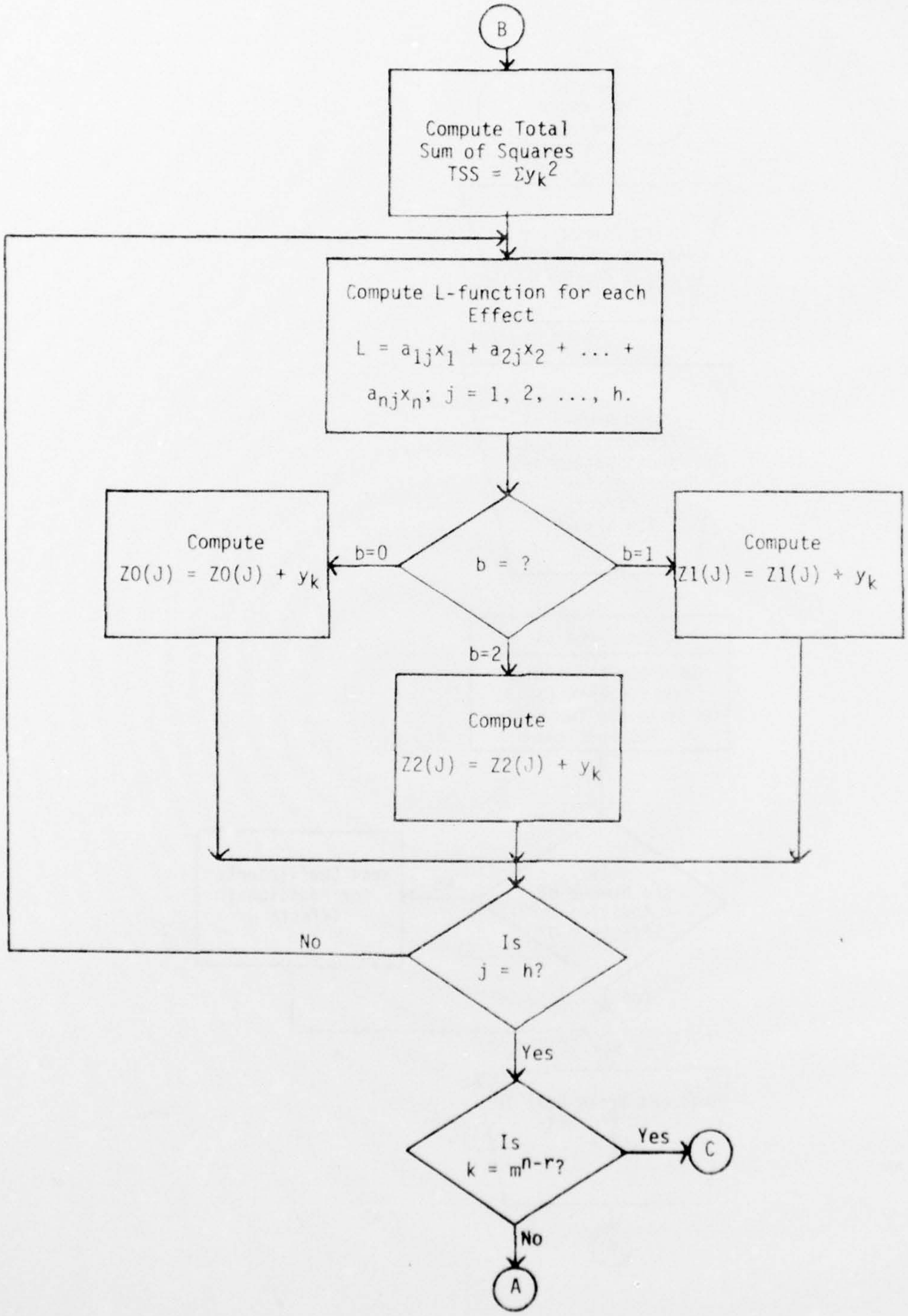
The desired analysis of variance for the one-third replicate of the 3^5 fractional factorial design is shown in Table 5 below.

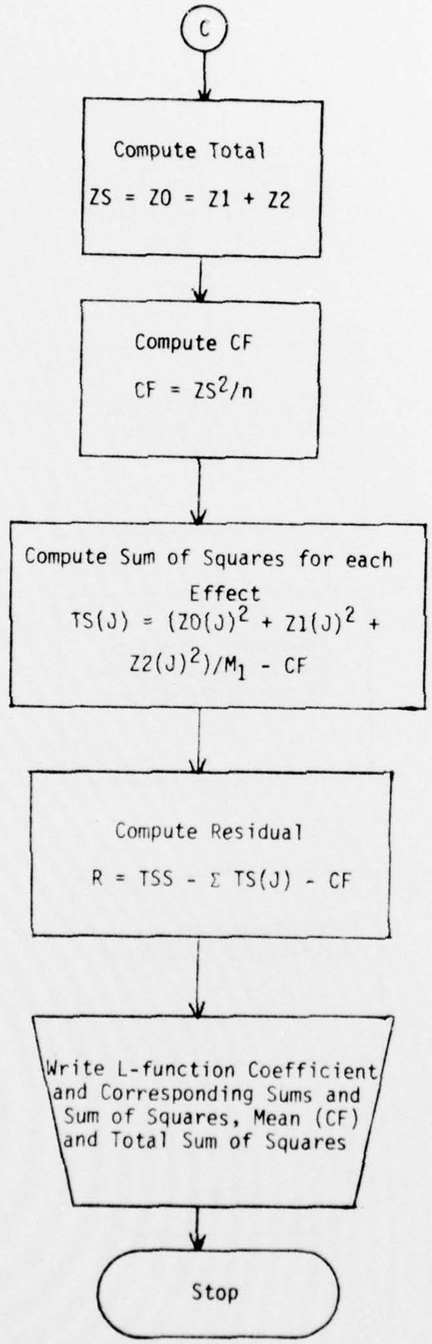
Table 5. ANOVA for the (1/3) x 3^5 Design

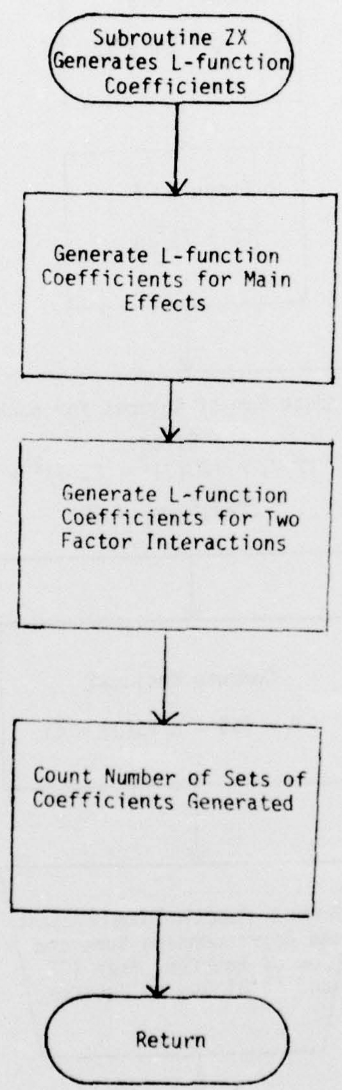
Source of Variation	Degrees of Freedom	Sum of Squares	Mean Square	F-ratio
A	2	1,137,460.22	586,730.11	30.13
B	2	4,732.07	2,366.04	<1.0
C	2	3,648,534.74	1,824,267.37	93.68
D	2	137,405.56	68,702.78	3.53
E	2	71,408.96	35,704.48	1.83
AB	4	901,368.15	225,342.04	11.57
AC	4	220,462.81	55,115.70	2.83
AD	4	26,786.67	6,696.67	<1.0
AE	4	120,065.93	30,016.48	1.54
BC	4	44,802.52	11,200.63	<1.0
BD	4	124,529.93	31,132.48	1.60
BE	4	141,847.41	35,461.85	1.82
CD	4	1,038,408.60	259,602.15	13.33
CE	4	35,385.63	8,846.41	<1.0
DE	4	151,467.04	37,866.76	1.94
R	30	584,209.33	19,473.64	
Total	80	8,388,875.56		

6. PROGRAM FLOW CHART




B
Compute Total
Sum of Squares
TSS = Σy_k^2
Compute L-function for each
Effect
L = a_1j x_1 + a_2j x_2 + ... +
a_nj x_n; j = 1, 2, ..., h.
b = ?
b=0
Compute
Z0(J) = Z0(J) + y_k
b=1
Compute
Z1(J) = Z1(J) + y_k
b=2
Compute
Z2(J) = Z2(J) + y_k
Is
j = h?
No
Yes
Is
k = m^n-r?
Yes
C
No
A


C
Compute Total
ZS = ZO = Z1 + Z2
Compute CF
$CF = ZS^2/n$
Compute Sum of Squares for each Effect
$TS(J) = (ZO(J)^2 + Z1(J)^2 + Z2(J)^2)/M_1 - CF$
Compute Residual
$R = TSS - \Sigma\ TS(J) - CF$
Write L-function Coefficient and Corresponding Sums and Sum of Squares, Mean (CF) and Total Sum of Squares
Stop


Subroutine ZX Generates L-function Coefficients
Generate L-function Coefficients for Main Effects
Generate L-function Coefficients for Two Factor Interactions
Count Number of Sets of Coefficients Generated
Return

7. PROGRAM LISTING

```
DG*MODANOVA.VAR
C   A SIMPLE ANALYSIS OF VARIANCE PROGRAM USING MODULO ARITHMETIC
C
         DOUBLE PRECISION Z0S,Z1S,Z2S,ZS,CF,TS,R
        INTEGER A, X, H
        DIMENSION Z0(475),Z2(475),Z1(475),Z0S(475),Z1S(475),Z2S(475)
        DIMENSION S(10),X(10),A(475,10),FMT(10),FM(10),TS(475)
          DATA S/'A','B','C','D','E','F','G','H','I','J'/
       READ(5,1)(FMT(I),I = 1,10)
        READ(5,1)(FM(I),I=1,10)
     1 FORMAT(10A6)
        DATA A/4750*0/
         R = 0
C
C READ NUMBER OF FACTORS,LEVELS, FRACTION AND NO. OF ADDITIONAL EFFECTS
C
        READ(5,3)N,M,NF,NEA
     3  FORMAT(8I5)
C
C  GENERATE L-FUNCTION COEFFICIENTS
C
        CALL  ZX(N,M,A,H)
         NZZ = H + 1
       H = H + NEA
        NE = (M**N)/NF
        M1 = NE/M
C
C CHECK FOR AND READ ADDITIONAL EFFECTS
C
         IF( NEA.EQ. 0)  GO TO 332
        DO 333  I = NZZ,H
        READ(5,4)(A(I,J),J=1,N)
     4  FORMAT(1X,10I1)
   333  CONTINUE
   332   DO 50 I = 1,NE
C
C READ CELL ID AND DATA VALUE
C
        READ(5,FMT)(X(IZ),IZ =1,N),Y
        WRITE(6,FMT)(X(IZ),IZ =1,N),Y
         TSS = TSS + Y*Y
```

```
      C
      C COMPUTE L-FUNCTION FOR EACH EFFECT
      C
            DO 200 J = 1,H
               L = 0
            DO 199 IZ = 1,N
            L=L+ (A(J,IZ)*X(IZ))
        199 CONTINUE
            L=MOD(L,M)
            IF(L-1)10,20,30
         10  Z0(J) = Z0(J)+Y
             GO TO 200
         20  Z1(J) = Z1(J)+Y
              GO TO 200
         30  Z2(J) = Z2(J)+Y
        200  CONTINUE
         50  CONTINUE
      C
      C COMPUTE TOTAL AND CORRECTION FACTOR
      C
            ZS = Z0(I) +Z1(1)+Z2(1)
            CF= ZS*( ZS /NF)
            WRITE(6,6)
              WRITE(6,7)(S(I),I=1,N)
          7 FORMAT(1X,10A1)
          6  FORMAT(1H1,14X,'0(MOD M)',6X,'1(MOD M)',6X,'2(MOD M)',2X,'SUM OF
           1SQUARES')
               DO 100 J=1,H
             Z0S(J)=Z0(J)*(Z0(J)/M1)
             Z1S(J)=Z1(J)*(Z1(J)/M1)
             Z2S(J)=Z2(J)*(Z2(J)/M1)
             TS(J)=    Z0S(J)+Z1S(J)+(Z2S(J) -CF)
             R = R + TS(J)
        100  CONTINUE
               WRITE(6,5)CF
           5 FORMAT(/1X,' MEAN',46X,G14.4)
               DO 101 J =1,H
               WRITE(6,FM)(A(J,I),I=1,N),Z0(J),Z1(J),Z2(J),TS(J)
           2 FORMAT(1H ,10I1,5X,4F14.4/)
        101  CONTINUE
             R = TSS-R-CF
              WRITE(6,9)R
           9 FORMAT(/1X,' R',49X,F14.4)
              WRITE(6,8)TSS
           8 FORMAT(/1X,' TOTAL',45X,G14.4)
                END
```

```
      SUBROUTINE ZX(N.M.A.H)
C
C     THIS PROGRAM CREATES THE COEFFICIENTS FOR THE L FUNCTION
C
      INTEGER A.H
      DIMENSION A(475 .10)
      MT=(N-1)*(M-1)
C
C GENERATE L-FUNCTION COEFFICIENTS FOR MAIN EFFECTS
C
      DO 300 J=1.N
      A(J.J)=1
  300 CONTINUE
        J =N+1
      MTT=J+MT-1
       N6 = N-1
C
C GENERATE L-FUNCTION COEFFICIENTS FOR TWO FACTOR INTERACTIONS
C
      DO 301 K1=1.N6
      DO 10  H=J.MTT
      A(H.K1)=1
   10  CONTINUE
      K2=K1+1
      H=J
       DO 11 I=K2.N
       M6 = M-1
      DO 12   II=1.M6
      A(H.I)=II
C
C COUNT NUMBER OF SETS OF COEFFICIENTS GENERATED
C
      H=H+1
   12  CONTINUE
   11  CONTINUE
       J=H
       MTT=H-1 +M6*(N-K2)
  301  CONTINUE
         H = H-1
      RETURN
      END
```

MOD-ANOVA: An Analysis of Variance Program for 2^n and 3^n Factorial Experiments

APPENDIX A
STUDY CONTRIBUTORS

1. Study Director

   Carl B. Bates, Methodology, Resources and Computation Directorate

2. Support Personnel

   Jerry Thomas, Methodology, Resources and Computation Directorate

MOD-ANOVA: An Analysis of Variance Program
for 2^n and 3^n Factorial Experiments

APPENDIX B
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